# INTERNATIONAL INDIAN SCHOOL - DAMMAM

## FIRST TERMINAL EXAMINATION- JUNE -2013

**CLASS : XII** **SET - A**

**SUBJECT: MATHEMATICS** **Max Marks : 100**

**Time: 3 hours**

***General Instructions:***

***1. All questions are compulsory.***

***2. The question paper consists of 29 questions divided into three sections A, B, C, Section A contains 10 questions of 1 mark each , Section B is of 12 question of 4 marks each and section C is of 7 questions of 6 marks each.***

***3. All questions in section A are to be answered in one word, one sentence or as per the requirement of the question.***

***4.There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.***

***5. Use of calculator is not permitted. You may ask for logarithmic tables, if required.***

## Section. A

1) Find the value for of $x$ and y , if $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$

2) Find the identity element in the set Q of all rational numbers for the binary operation *

defined by $a*b = \frac{ab}{5} \forall a, b \in Q$.

3) Find the value of $\cos\left(\sec^{-1} x + \operatorname{cosec}^{-1} x\right), |x| \geq 1$.

4) Evaluate $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

5) If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$.

6) If $f : R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f[f(x)]$.

7)Differentiate $\sin\left[\log\left(x^3 - 1\right)\right]$, w.r.t.x.

8) Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.

9)The radius of a circle is increasing uniformly at the rate of 3cm/s. Find the rate at which the area of the circle is increasing when the radius is 10cm.

10) Examine the continuity of the function $f(x)=x^2+5$ at $x=-1$

## Section. B

11) If $f:R\to R$ be defined by $f(x)=3x+7$. Show that $f$ is invertible and hence find $f^{-1}$.

12) Find the values of $x$, if $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$.

13) Show that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

**OR**

Using properties of determinants show that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

14) Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\ ,\quad -\frac{1}{\sqrt{2}}\le x\le 1.$

**OR**

Express in the simplest form $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

15) Let $A=\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and 'I' be the identity matrix of order 2 . Show that

$$I+A=I-A\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}.$$

16) Differentiate $\sin^{-1}\frac{2^{x+1}}{1+4^x}$ .with respect to x

17) Find the intervals in which the function $f$ given by $f(x)=\sin x+\cos x, 0\le x\le 2\pi$ is strictly increasing or strictly decreasing.

**OR**

Find the equation of the tangent to the curve $y=\dfrac{x-7}{(x-2)(x-3)}$ at the point where it cut the X-axis.

18) A store in a mall has three dozen shirts with 'SAVE ENVIRONMENT' printed, two dozen ' SAVE TIGER' printed and five dozen shirts with 'GROW PLANTS' printed .The cost of each shirt is Rs. 595, Rs.610 and Rs795 respectively. All these items were sold in a day. Find total collection of the store using matrix method. Which shirt you would like to buy and why ?

19) If $y=e^{a\cos^{-1}x}, -1\le x\le 1,$ show that $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}-a^2y=0$

20) If $A=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$ show that $A^2-5A-14I=0$. Hence find $A^{-1}$.

21) If $\tan^{-1}\left(\dfrac{x-1}{x-2}\right)+\tan^{-1}\left(\dfrac{x+1}{x+2}\right)=\dfrac{\pi}{4}$, find the value of $x$.

22) Show that $f(x)=|x-3|\,\forall x\in R$, is continuous but not differentiable at x=3.

**OR**

If $x=a\left(\cos t+\log\tan\dfrac{t}{2}\right), y=a\sin t$ fnd $\dfrac{d^2y}{dx^2}$.

## Section -C

23) Let $A=Q\times Q$ and let * be a binary operation on A defined by

$(a,b)*(c,d)=(ac,b+ad);\forall(a,b),(c,d)\in A$, then with respect to * on A

(i) Is * commutative (ii) Is * associative

(iii) Find the identity element of A (iv)Find invertible element of A

24) Find the product of matrices $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}, B=\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ and use it for solving the equations $x-y+2z=1$ ; $2y-3z=1$ ; $3x-2y+4z=2$.

**OR**

Using elementary transformations find the inverse of the matrix $\begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix}$

25) Two sign boards, one circular and one square are to be made using a wire of length 40 m and cutting it into two pieces. The sign boards are to depict "BE HONEST" and " BE PUNCTUAL" and these are to be displayed near the main gate of the school. What should be the lengths of the two pieces , so that the combined area of the square and the circle is minimum? Do you think these values are important in life and why?

26) If $\tan\left(\frac{x^2-y^2}{x^2+y^2}\right)=a$ then prove that $\frac{dy}{dx}=\frac{y}{x}$.

27) Differentiate with respect to $x$ $\quad (x\cos x)^x+(x\sin x)^{\frac{1}{x}}$

28) Show that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

29) Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex at one end of the major axis.

**OR**

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.