INTERNATIONAL INDIAN SCHOOL , DAMMAM
FIRST TERMINAL EXAMINATION 2014
GRADE – 12
SUBJECT : MATHEMATICS

TIME : 3 HOURS **Max. Marks : 100**

SET – A

General Instructions

1.All questions are compulsory.

2.The question paper consists of **26** questions divided into sections **A, B** and **C**. Section **A** comprises of **6** questions of one mark each, Section **B** comprises of **13** questions of four marks each and Section **C** comprises of **07** questions of six marks each.

3.All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4.There is no overall choice. However, internal choice has been provided in **04** questions of four marks each and **02** questions of six marks each. You have to attempt only one of the alternatives in all such questions.

5.Use of calculator is not permitted. You may ask for logarithmic tables , if required.

6. This paper consists of FOUR printed pages.

SECTION A

1. Evaluate $\tan^{-1}\left[2\, cos(2\sin^{-1}(\frac{1}{2}))\right]$

2. If f : R→ R, $f(x) = \sqrt[3]{(3 - x^3)}$ then find f o f (x).

3. Write the value of x + y + z ,if $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

4. For what of k, the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ *is singular*

5. If $y = (\sin x + \cos x)^2$, *find* $\frac{dy}{dx}$ *at* $x = \frac{\pi}{4}$.

6. Examine the continuity of $f(x) = \frac{2}{x-5}$ at $x \neq 5$.

SECTION B

7. Let Z be the set of all integers and R be the relation on Z defined as R = { (a , b) : a ,b ∈ Z and (a – b) is divisible by 5 }.Prove that R is an equivalence relation.

8. Define a binary operation * on set { 0,1,2,3,4,5} as $a*b = \begin{cases} a+b\ , \ if\ a+b<6 \\ a+b-6, if\ a+b\ \geq 6 \end{cases}$ show that zero is the identity for this operation and each element a≠0 of the set is invertible with (6- a) being the inverse of a.

OR

Let f : N→R be a function defined as $f(x)= 4x^2 + 12x +15$.show that f: N→S where S is the range of f , is invertible .find inverse of f .

9. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$ in simplest form.

10. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^n I + n a^{n-1} b A$,where I is the identity matrix of order 2 and n∈N .

11. A ladder 5 m long is leaning against a wall . The bottom of the ladder is pulled along the ground ,away from the wall,at the rate of 2 cm/sec.How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

12. Using differentials to approximate $(25)^{\frac{1}{3}}$.

OR

Verify Lagrange's Mean value theorem for the function $f(x) = x + \frac{1}{x}$ in [1 , 3] .

13. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$,show that $A^2 - 5A + 7I = O$. Hence find inverse of A.

14.Prove that ,using properties of determinants

$$\begin{vmatrix} X^2+1 & XY & XZ \\ XY & Y^2+1 & YZ \\ ZX & ZY & Z^2+1 \end{vmatrix} = 1 + X^2 + Y^2 + Z^2.$$

15. If $y = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ *show that* $\frac{dy}{dx} = \sec x$. Also find the value of $\frac{d^2y}{dx^2}$ $at\ x = \frac{\pi}{4}$.

16. Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1$ is an increasing function of x throughout its domain.

OR

Find the intervals in which $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing or decreasing.

17. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t\cos t)$ find $\frac{d^2y}{dx^2}$.

OR

If $y = \cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ find $\frac{dy}{dx}$

18. Find the equations of the tangent and normal to the curve $y^2 = 3x - 2$ which is parallel to the line $4x - 2y + 5 = 0$.

19. Find $\frac{dy}{dx}$, if $\sin^2 y + \cos(xy) = k$.

SECTION C

20 If $x^{13} y^7 = (x+y)^{20}$ find $\frac{dy}{dx}$

OR

Differentiate with respect to x, $(x\cos x)^x + (\sin x)^{\cos x}$.

21 Show that the height of the cylinder of maximum volume that can be inscribed in a right circular cone of height h and semi vertical angle β is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \beta$.

OR

Show that semi- vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}(1/3)$.

22 If $y = e^{a\cos^{-1}x}$, $-1 \leq x \leq 1$,show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2 y = o.$$

23 Show that the function $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x - 4, & x > 2 \end{cases}$ is continuous at $x = 2$,but not differentiable.

24 A school wants to award its students for the values honesty, regularity and hard work with a total cash award of Rupees 6000.Three times the award money for hardwork added to that given for honesty amounts to Rupees 11000.The award money given for honesty and hardwork together is double the one given for regularity. Represent the above situation algebrically and find the award money for each value ,using matrix method.Apart from these values suggest one more value which the school must include for awards.

25 Find the inverse of the following matrix using elementry operations.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

26 Solve the equation :

$$\tan^{-1}(x - 1) + \tan^{-1}x + \tan^{-1}(x + 1) = \tan^{-1}3x.$$
