INTERNATIONAL INDIAN SCHOOL – DAMMAM
FIRST TERMINAL EXAMINATION JUNE - 2015

GRADE XII

Subject : Mathematics **Max marks : 100**
Time : 3 hours

SET A

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of 26 questions divided into three sections A, B and C*
3. *Section A contains 6 questions of 1 mark each, Section B contains 13 questions of 4 marks each, Section C contains 7 questions of 6 marks each.*
4. *There is no overall choice in the paper. However, internal choice is provided in four questions of 4 marks each and 2 questions of 6 marks each.*
5. *Use of calculators is not permitted.*

SECTION A

Questions from 1 to 6 carry 1 mark each

1. Find the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

2. If $f : R \rightarrow R$ and $g : R \rightarrow R$ is given by $f(x) = \sin x$ and $g(x) = 2x^3$, find $g \circ f$ and $f \circ g$.

3. If $A = \begin{bmatrix} 2 & \alpha & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then find the value of α for which A^{-1} does not exist.

4. Evaluate $\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix}$

5. State the points of discontinuity for the function $f(x) = [x]$, in $-3 < x < 3$

6. Find the second derivative of $\log x$ with respect to x.

SECTION B

Questions from 7 to 19 carry 4 marks each

7. Show that the relation R in the set Z of integers given by $R = \{(a, b): 2 \text{ divides } a - b\}$ is an equivalence relation.

8. Solve the following for **x.**

$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$$

OR

Find the value of x, if $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

9. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, Verify that $A^2 - 4A - 5I = 0$ and hence find the inverse.

10. Prove that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \cos^{-1}\frac{36}{85}$

11. Find the value of **a** and **b** if the function $f(x) = \begin{cases} 3ax + b, if\ x > 1 \\ 11, if\ x = 1 \\ 5ax - 2b, if\ x < 1 \end{cases}$ is continuous at $x = 1$

12. Let A = Q X Q and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$ $\forall (a, b), (c, d) \in A$. Show that $*$ is commutative and associative. Find the identity element of A.

OR

Define a binary operation $*$ on the set $\{0, 1, 2, 3, 4, 5\}$ as

$$a * b = \begin{cases} a + b, & if\ a + b < 6 \\ a + b - 6, & if\ a + b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element $a \neq 0$ is invertible with $6 - a$ being the inverse of a.

13. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, Show that $(aI + bA)^n = a^n I + na^{n-1}bA$

14. If $x = a \sin t$ and $y = a\left[\cos t + \log\left(\tan\frac{t}{2}\right)\right]$, find $\frac{d^2y}{dx^2}$.

OR

Differentiate $f(x) = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ w. r. t. x.

15. Show that $f : N \rightarrow N$ given by

$$f(x) = \begin{cases} x + 1, & if\ x\ is\ odd, \\ x - 1, & if\ x\ is\ even \end{cases}$$ is both one-one and onto.

16. Express the following matrix as the sum of a symmetric and a skew symmetric matrix

$$\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$$

17. Prove that $\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2 b^2 c^2$

OR

Prove that $\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$

18. If $\log (x^2 + y^2) = 2 \tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx} = \frac{x+y}{x-y}$

19. If $y = \sin^{-1}x$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

SECTION C

Questions from 20 to 26 carry 6 marks each

20. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening.

21. . Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

$$x - y + 2z = 1$$
$$2y - 3z = 1$$
$$3x - 2y + 4z = 2$$

OR

Using elementary transformations, find the inverse of the following matrix

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

22. Consider $f : R \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible. Find the inverse of f.

23. Differentiate w. r. t. x, $y = (\cos x)^x + (\sin x)^{\frac{1}{x}}$

24. A school wants to award its students for the values of Honesty, Regularity and Hardwork with a total cash cash award of Rs. 6000. Three times the award money for Hardwork added to that given for Honesty amounts to Rs. 11,000. The award money given for Honesty and Hardwork together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hardwork, suggest one more value which the school must include for awards.

25. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

OR

Show that the semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

26. Prove that $\tan^{-1}\left(\frac{cosx}{1+sinx}\right) = \frac{\pi}{4} - \frac{x}{2}$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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