INTERNATIONAL INDIAN SCHOOL - DAMMAM

MODEL EXAMINATION- JANUARY -2018

CLASS : XII **SET - A**
SUBJECT: MATHEMATICS **Max Marks : 100**
Time: 3 hours

General Instructions:

1. All questions are compulsory.

2. The question paper consists of 29 questions divided into four sections A, B, C, D ,Section A contains 4 questions of 1 mark each , Section B is of 8 question of 2 marks each section C is of 11 questions of 4 marks each and Section D is of 6 questions of 6 marks each.

3. Use of calculator is not permitted. You may ask for logarithmic tables, if required.

Section. A

1) Write the value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

2) Find $\frac{dy}{dx}$ if $x = 2at^2$, $y = at^4$

3) Evaluate $\int \frac{dx}{x(1+logx)}$

4) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non- zero vectors such that $\vec{a} . \vec{b} = 0$ and $\vec{a} + \vec{b} = \vec{c}$, then write the relation between their magnitudes.

Section. B

5) Differentiate $\tan^{-1}\sqrt{1+x^2} - x$.

6) Evaluate $\int \frac{(x-4)}{(x-2)^3} \; e^x \, dx$

7) Find the equation of the curve passing through the point $(0, -2)$ given that at any point (x, y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point.

8) If $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$, then show that the angle between $\vec{a}$ and $\vec{b}$ is 60°.

9) A can solve 90% of the problems given in a book and B can solve 70%. What is the probability that at least one of them will solve the problem, selected at random from the book.

10) Find the equation of the line joining (1,2) and (3,6) using determinant.

11) Evaluate $\int_0^1 \frac{dx}{\sqrt{2x+3}}$.

12) Use differentials to approximate $\sqrt{25.2}$.

Section -C

13) Using properties of determinants prove that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca).$$

14) If $(tan^{-1}x)^2 + (cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then find x.

OR

Solve $cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$

15) Evaluate $\int \sqrt{tan\theta} + \sqrt{cot\theta}\ d\theta$.

16) If $y = log\left(x+\sqrt{x^2+a^2}\right)$ prove that $(x^2+a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

OR

Separate the interval $\left[0,\frac{\pi}{2}\right]$ into subintervals in which the function $f(x) = sin^4x + cos^4x$ is strictly increasing or strictly decreasing.

17) Find the general solution of the differential equation $\frac{dy}{dx} - y = \sin x$

18) Prove that $\vec{a}.\left[(\vec{b}\times\vec{c})\times(\vec{a}+2\vec{b}+3\vec{c})\right] = [\,\vec{a}\ \vec{b}\ \vec{c}\,]$

19) Find the particular solution of the differential equation

$$2y\, e^{\frac{x}{y}}\, dx + \left(y - 2x\, e^{\frac{x}{y}}\right) dy = 0 \text{ , given that } x = 0\,, y = 1.$$

20) Evaluate $\int (3x-2)\sqrt{x^2+x+1}\; dx$

OR

Evaluate $\int_0^1 cot^{-1}(1-x+x^2)\, dx$

21) A bag I contains 5 red and 4 white balls and a bag II contains 3 red and 3 white balls. Two balls are transferred from bag I to bag II and then one ball is drawn from bag II. If the ball drawn from bag II is red, then find the probability that one red and one white ball are transferred from the bag I to bag II.

22) Find the coordinates of the point where the line through the points $(3,-4,-5)$ and $(2\,,-3\,,1)$ crosses the plane $2x+y+z=7$.

23) The probability that a randomly selected voter will vote for party A is 0.2 and the probability that he vote for party B is 0.5, otherwise he will vote for independent parties. What is the probability that out of 6 voters, 3 or more will vote for party B.

Section -D

24) Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{\jmath} - 2\hat{\jmath} + 3\hat{k}) - 4 = 0$ and $\vec{r}.(-2\hat{\imath} + \hat{\jmath} + \hat{k}) + 5 = 0$ and whose intercept on X axis is equal to that of on Y axis.

OR

Find the vector equation of the plane passing through the points $(3,4,2)$ and $(7,0,6)$ and perpendicular to the plane $2x - 5y - 15 = 0$ .Also show that the plane thus obtained contains the line $\vec{r} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k} + \lambda\,(\hat{\imath} - \hat{\jmath} + \hat{k})$

25) Consider $f : R_+ \to [-9,8]$ given by $(x) = 5x^2 + 6x - 9$. prove that f is invertible with

$$f^{-1}(y) = \frac{\sqrt{54+5y} - 3}{5}$$

26) Find the area of the region bounded by the curves $x = at^2$ and $y = 2at$ between the ordinate corresponding to $t = 1\ and\ t = 2$

OR

Using integration find the area of the region bounded by the curve $y = \sqrt{5 - x^2}$ and $y = |x - 1|$.

27) The management committee of a residential colony decided to award some of its members for honesty , some for helping others and some others for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of the awardees for supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others. Using matrix method , find the number of awardees of each category . Apart from these values suggest one more value which the management of the colony must include for awards

28) Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

OR

A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is $2cm$ and volume is $8m^3$. If building of tank costs Rs 70 per sq.m for the base and Rs.45 per sq.m for sides. What is the cost of least expensive tank?

29) A dealer in a rural area wishes to purchase some sewing machines. He has only Rs. 57,600 to invest and has space for at most 20 items. An electronic item costs him Rs.3,600 and a manually operated machine costs Rs.2,400. He can sell an electric machine at a profit of Rs. 220 and a manually operated machine at a profit of Rs.180.Assuming that he can sell all machines that he buys, how should he invest his money in order to maximize his profit? Make it as an LPP and solve it graphically.

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3. Use of calculator is not permitted. You may ask for logarithmic tables, if required.

Section. A

1) Evaluate $\int \frac{dx}{x(1+logx)}$

2) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non- zero vectors such that $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} + \vec{b} = \vec{c}$, then write the relation between their magnitudes.

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Section. B

5) Find the equation of the line joining (1,2) and (3,6) using determinant.

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OR

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