INTERNATIONAL INDIAN SCHOOL-DAMMAM
PRELIMINARY EXAMINATION- FEB. 2013

CLASS: XII **SUBJECT: MATHEMATICS**

TIME: 3.00 HRS. **MAX. MARKS: 100**

SET-A

General instructions:

(i) All questions are compulsory.
(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one(1) mark each, section B comprises of 12 questions of four(4) marks each and section C comprises of 07 questions of six(6) marks each.
(iii) All questions of section A are to be answered in one word, one sentence or as per the exact requirement of the question.
(iv) There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculator is not permitted. You may ask for logarithmic tables, if required.

SECTION- A

1. Evaluate : $\int \frac{sec^2(1+\log x)}{x}\, dx.$

2. Evaluate : $\int_{-\pi/4}^{\pi/4} x^3\, sin^4 x\ dx.$

3. If $\int \left(\frac{x-1}{x^2}\right) e^x\ dx = f(x)\, e^x + C$, then write the value of f(x).

4. Find integrating factor of the linear differential equation, $(x+y)\frac{dy}{dx} = 1.$

5. Form the differential equation corresponding to the function $y = ax.$

6. Write the projection of the vector $\hat{\imath} - \hat{\jmath}$ on the vector $\hat{\imath} + \hat{\jmath}$.

7. Write the equation of st. line passes through (2, 0, -1) and perpendicular to the plane 3x – y + 2z = 8.

8. Find the distance of the plane $3x - 3y + 3z = 0$ from the point $(-2, 3, -1)$.

9. Give an example of a function which is continuous at $x = 1$, but not differentiable at $x = 1$.

10. Find k, $if \ f(x) = \begin{cases} \frac{x^2-25}{x-5}, & if \quad x \neq 5 \\ k, & if \quad x = 5 \end{cases}$ is continuous at $x=5$.

SECTION- B

11. Consider $f : R^+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$.

OR

Let A=$\mathbb{Z} \times \mathbb{Z}$ and * be a binary operation on A defined by (a, b) * (c, d) = (a+c, b+d). Show that * is commutative and associative. Also, find the identity element for * on A, if any.

12. Solve for x : $cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$.

13. If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$, find A.B and use it to solve the following equations :

$$x - y + 2z = 1, \quad 2y - 3z = 1, \quad 3x - 2y + 4z = 2 .$$

14. Find inverse of A, by elementary row transformations, where $A = \begin{bmatrix} 5 & -6 \\ -2 & 3 \end{bmatrix}$.

15. If $y = \left[\log\left(x + \sqrt{x^2+1}\right)\right]^2$, then prove that,

$$(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 2 = 0 .$$

16. If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$, find the value of $\frac{dy}{dx}$ $at\ \theta = \frac{\pi}{4}$.

17. Evaluate: $\int \frac{x+2}{\sqrt{(x-2)(x-3)}} dx.$

18. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

OR

Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2)\,(x-3)}$, at the point, where it cuts the x- axis.

19. Evaluate : $\int_{-3}^{2} (\,|x+3| + |x-1|\,)\, dx.$

OR

Evaluate the integral using limit of sums : $\int_1^4 (2x^2 - x)\, dx.$

20. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, prove that the vectors given by $\vec{p} = \vec{a} + \vec{b}$, $\vec{q} = \vec{b} + \vec{c}$, $\vec{r} = \vec{c} + \vec{a}$ are also coplanar.

21. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

OR

Find the distance of the point (-3, 1, -4) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{2-z}{-2}$ and the plane $\vec{r}\,.\left(\hat{\imath} - \hat{\jmath} + \hat{k}\right) = 5.$

22. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) P(X > 6) (iii) P (0 < X < 3) .

SECTION- C

23. If $x \neq y \neq z$ and $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = 0$. Prove that $xyz = \frac{-1}{p}$.

24. Show that volume of greatest cylinder which can be inscribed in cone of height h and semi vertical angle α is $\frac{4}{27}\pi h^3 tan^2\alpha$.

25. Using integration, find the area of the region :
$\{(x, y) : 9x^2 + y^2 \leq 36 \; and \; 3x + y \geq 6\}$.

OR

Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

26. Solve the differential equations :
$\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0.$

OR

$(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$

27. Find the equation of the plane passing through the points (2, 1, -1) and (-1, 3, 4) and perpendicular to the plane x – 2y + 4z =10. Also show that the plane thus obtained contains the line:
$\vec{r} = -\hat{\imath} + 3\hat{\jmath} + 4\hat{k} + \mu\,(3\hat{\imath} - 2\hat{\jmath} - 5\hat{k})$.

28. A retired person has Rs. 70,000 to invest in two type of bonds. First type of bond yields an annual income of 8% on the amount invested and the second type of bond yields 10% per annum. As per norms he has to invest minimum of Rs. 10,000 in first type and not more than Rs. 30,000 in second type. How should he plan his investment so as to get maximum return after one year of investment ? Do you think that a person should start saving at an early age for his retirement ? Can you name some avenues ?

29. In answering a question on a multiple choice questions test with four choices per question. A student knows the answer, guesses or copies the answer. If ½ be the probability that he knows the answer, ¼ be the probability that he guesses it and ¼ that he copies it. Assuming that a student who copies the answer will be correct with the probability ¾ . What is the probability that the student knows the answer given that he answered it correctly ?
"Rahul" does not know the answer to one question in the test. The evaluation process has negative marking. Which value would "Rahul" violate if he restores to unfairness ? How would his personality would be hampered ?

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