

INTERNATIONAL INDIAN SCHOOL – DAMMAM

PRELIMINARY EXAMINATION

TOTAL MARKS: 100

CLASS XII - MATHEMATICS

TIME: 3 HOURS

SET A

General Instructions

1. All Questions are compulsory
2. Question paper consists of 29 questions divided into three sections A,B,C. Section A comprises of 10 questions of one mark each. Section B comprises of 12 questions of four marks each and section C consists of seven questions of six marks each.
3. There is no overall choice. Internal choice is given in 4 questions of 4 marks each 2 questions of 6 marks each. Attempt only one of the alternatives in such questions

Section - A (Questions 1 to 10 carry 1 mark each)

1. If the binary operation * defined on \mathbb{R} of real numbers is defined as $a*b = \frac{3ab}{7}$ write the identity element in \mathbb{R} for *

2. Write the value of the following $\tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{a-b}{a+b}\right)$

3. For what value of k, the matrix $\begin{bmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{bmatrix}$ is skew symmetric?

4. Find the value of x+y if $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

5. Evaluate $\int e^{3\log x} x^4 dx$.

6. Find the equation of a line passing through (1, -1, 0) and parallel to the line

$$\frac{x-2}{3} = \frac{2y+1}{2} = \frac{5-z}{1}$$

7. If A is square matrix of order three such that $|adj A| = 225$. Find $|A^T|$

8. If the vectors $2\hat{i} - \hat{j} + \lambda\hat{k}$, $\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$ are coplanar, find the value of λ .

9. Evaluate $\int \frac{\sec^2 x}{\operatorname{Cosec}^2 x} dx$.

10. Find the area of parallelogram whose diagonals are $2\vec{i} - 3\vec{j} + 4\vec{k}$ and $-3\vec{i} + 4\vec{j} - \vec{k}$

Section - B (Questions 11 to 22 carry 4 marks each)

11. Solve for x

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) - \frac{1}{2}\tan^{-1}x = 0, \quad x > 0$$

12. Find the intervals in which the function given by $f(x) = \sin x + \cos x$, where $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

OR

Show that the curves $x = y^2$ and $xy = k^2$ cut orthogonally, if $8k^2 = 1$

13. Without expanding the determinant, show that $(a+b+c)$ is a factor of the following determinant

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

14. If the lines $\frac{x-1}{-3} = \frac{y-2}{-2k} = \frac{z-3}{2}$ and $\frac{x-1}{k} = \frac{y-2}{1} = \frac{z-3}{5}$ are perpendicular, find the value of k.

Hence find the equation of plane containing these lines.

15. Show that the differential equation $xdy - ydx = \sqrt{x^2 + y^2}dx$ is homogeneous and solve it.

16. Find the particular solution of the differential equation $(x - \sin y)dy + (\tan y)dx = 0$ given $y=0$ when $x=0$

OR

Form the differential equation representing the family of ellipses having foci on X axis and centre at origin.

17. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, express $\vec{\beta}$ in the form $\vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

18. Find the equation of tangent lines to the curve $y = 4x^3 - 3x + 5$ which are perpendicular to the line $9y + x + 3 = 0$.

19. If $x^p y^q = (x + y)^{p+q}$ Prove that $\frac{dy}{dx} = \frac{y}{x}$

OR

If $y = \tan^{-1} \left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right)$ $0 < |x| < 1$ Find $\frac{dy}{dx}$.

20. Evaluate $\int \frac{x^3 + x + 1}{x^2 - 1} dx$

OR

Evaluate $\int \frac{2 + \sin x}{1 + \cos x} e^{\frac{x}{2}} dx$

21. There is a group of 50 people who are patriotic out of which 20 believe in non violence. Two persons are selected at random out of them, write the probability distribution for the selected persons who are non violent. Also find the mean of the distribution. Explain the importance of non violence in patriotism.

22. Prove that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

Section – C (6 Marks each)

23. Find the distance of the point **(1,-2,3)** from the plane $x - y + z = 5$ measured parallel to the

line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$

OR

Find the equation of the perpendicular drawn from the point (2, 4, -1) to the

line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$

24. Make a rough sketch of the region given below and find its area using integration $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

25. Urn A contains 1 white 2 black, and 3 red balls. Urn B contains 2 white, 1 black and 1 red ball. Urn C contains 4 white, 5 black and 3 red balls. One urn is chosen at random and two balls are drawn. These happen to be one white and one red. What is the probability that they come from Urn A.

26. Two institutions decided to award their employees for the three values of resourcefulness, competence, and determination in the form of prizes at the rate of ₹ x, ₹ y, ₹ z respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with a total prize money ₹37,000 and second institution decided to award respectively 5, 3, and 4 employees with a total prize money of ₹ 47,000. If all the three prizes taken together amount to ₹ 12,000. Using matrix method find the values of x, y, z. What values are described in this question.

27. Show that the volume of the greatest cylinder that can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

OR

Show that height of the closed cylinder of given surface and maximum volume is equal to diameter of its base.

28. A company produces soft drinks that has a contract which requires that a minimum of 80 units of chemical A and 60 units of the chemical B to go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs ₹ 10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B that costs ₹ 4. How many packets of mixes from S and T should the company purchase to honour the contract requirement and yet minimize the cost. Make a LPP and solve graphically.

29. Evaluate $\int (x+1)\sqrt{1-x-x^2} dx$
