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INTERNATIONAL INDIAN SCHOOL – DAMMAM

PRELIMINARY EXAMINATION – JAN/FEB- 2015

CLASS: XII

SUBJECT: MATHEMATICS

MAX. MARKS: 100

TIME: 3 Hrs.

SET – A

INSTRUCTIONS:

(i) *All questions are compulsory.*

(ii) *Question number 1 to 6 carry 1 mark each. Question number 7 to 19 carry 4 marks each. Question number 20 to 26 carry 6 marks each.*

(iii) *There is no overall choice but internal choices have been provided in four questions of 4 marks and in the two questions of 6 marks.*

(iv) *Use of calculator is not permitted, however you may use logarithmic or trigonometric tables.*

SECTION – A

Qn. 1. If $\boldsymbol{R}$ is a relation defined on N given by $\boldsymbol{R} = \{(a, b):\ a + b \ is\ even\ number\}$. Verify transitive property.

Qn. 2. If $y = \sin^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$ then find $\frac{dy}{dx}$.

Qn. 3. Evaluate : $\int_0^{\pi/2} \log\ tanx\ dx.$

Qn. 4. Find a vector of magnitude 9 in the direction of the vector $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}.$

Qn. 5. Write the condition for three vectors $\vec{a} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k},\quad \vec{b} = a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}$ $and\quad \vec{c} = a_3\hat{\imath} + b_3\hat{\jmath} + c_3\hat{k}$ to be the coplanar.

Qn. 6. What is the integrating factor in the differential equation $2x^2\frac{dy}{dx} - 2xy + y^2 = 0.$

SECTION –B

Qn. 7. Let A = Z x Z and * be a binary operation on A defined by (a, b) * (c, d) = (a+c, b+d). Show that * is commutative and associative. Find identity element for * on A. Also discuss about the inverse of a.

OR

If $f\ (x) = \frac{4\,x+3}{6\,x-4}$, $x\ \neq 2/3$. Show that $fof\ (x) = x$ for all $x\ \neq 2/3$. What is the inverse of f ?

Qn. 8. Prove that $tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\,cos^{-1}x, -\frac{1}{\sqrt{2}} \leq x\ \leq 1.$

Qn. 9. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) $P(X < 3)$ (iii) $P(X > 6)$ (iv) $P(0 < X < 3)$.

Qn. 10. Prove the following, using properties of determinants:

$$\begin{vmatrix} a + bx^2 & c + dx^2 & p + qx^2 \\ ax^2 + b & cx^2 + d & px^2 + q \\ u & v & w \end{vmatrix} = (x^4 - 1)\begin{vmatrix} b & d & q \\ a & c & p \\ u & v & w \end{vmatrix}.$$

Qn. 11. By using elementary transformations, prove that $A^{-1} = \frac{1}{19} A.\ where\ A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}.$

Qn. 12. Find the value of k, for which the function

$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & where \quad -1 \le x < 0 \\ \frac{2x+1}{x-1} & where \quad 0 \le x < 1 \end{cases}$$ is continuous at $x = 0$.

Qn. 13. If $x = a(\cos t + t\sin t)\ and\ y = a(\sin t - t\cos t),\ where\ 0 < t < \frac{\pi}{2},$

$find\ \frac{d^2y}{dx^2}.$

OR

If $y^x = e^{y-x},\ then\ prove\ that\ \frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$.

Qn. 14. Find the equation of tangent to the curve $y = x^3$ at which the slope of the tangent is equal to y-coordinate of the point.

Qn. 15. Evaluate: $\int \frac{x+2}{\sqrt{(x-2)(x-3)}}\, dx.$ OR $\int x^2 \log(x+1)\, dx$.

Qn. 16. Evaluate: $\int \frac{3x+5}{x^3-x^2-x+1}\, dx.$

Qn. 17. Evaluate: $\int_1^4 (\,|x-1| + |x-2| + |x-4|\,)\, dx.$

Qn. 18. Find the distance of the point $(-2, 3, -4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured Parallel to the plane $4x + 12y - 3z + 1 = 0.$

Qn. 19. If vectors $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\vec{a} + \mu\,\vec{b}$ is perpendicular to $\vec{c}$, then find the value of μ.

OR

If $\vec{\alpha} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{\beta} = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$ then express $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$, where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$.

SECTION – C

Qn. 20. Solve the differential equation,

$$\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}y\,dx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}x\,dy.$$

Qn. 21. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods X and Y are available at a cost of Rs. 4 and Rs. 3 per unit respectively. One unit of the food X contains 200 units of vitamins, 1 unit of minerals and 40 calories, whereas one unit of food Y contains 100 units of vitamins, 2 unit of minerals and 40 calories. Find what combination of X and Y should be used to have least cost, satisfying the requirements? Explain how fast foods are harmful for our health ?

Qn. 22. There are three coins. One is a two headed coin another is a biased coin that heads 75% of the times and the third is an un biased coin. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the biased coin ?

Qn. 23. Find the equation of plane passes through three non collinear points (1, 1, -2), (2, -1, 1), (1, 2, 1). Also find the coordinates of the point of intersection of this plane and the line $\frac{x-3}{2} = \frac{y+1}{-2} = \frac{z+1}{1}$.

Qn. 24. Show that the height of the closed right circular cylinder of given surface and maximum volume is equal to the diameter of the base.

OR

An open box, with a square base, is to be made out of a given quantity of metal sheet of area c^2. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$.

Qn. 25. Evaluate the area under the curve $y = |x + 3|$ above the X – axis and between x= - 6 and x= 3.

OR

Find the area of the region included between the parabola $y = \frac{3x^2}{4}$ and the line $3x - 2y + 12 = 0$

Qn. 26. 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises "Hard workers", the second group has "Honest and Law abiding" students and the third group contains "Vigilant and Obedient" students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of first and second group is four times that of the third group. Using Matrix method, find the number of students in each group. Apart from the values, "Hard work", "Honesty and Respect for Law", "Vigilance and Obedience" suggest one more value, which in your opinion, the school should consider for awards and why ?

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Qn. 5. Write the condition for three vectors $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ and $\vec{c} = a_3\hat{i} + b_3\hat{j} + c_3\hat{k}$ to be the coplanar.

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