INTERNATIONAL INDIAN SCHOOL DAMMAM

PRELIMANARY EXAMINATION – 2016

CLASS:- 12 **SET A** TIME : 3 HOURS

SUBJECT:- MATHEMATICS MAX. MARKS : 100

General Instructions:

a) *All questions are compulsory*

b) *The question paper consists of 26 questions divided into sections A, B and C. Section A consists of 6 questions of one mark each, section B consists of 13 questions of four marks each and section C consists of 7 questions of six marks each.*

c) *There are no overall choices. However internal choices has been provided in 4 questions of four marks each and two questions of six marks each. You have to attempt only one of the alternatives in all such questions.*

d) *Use of calculator is not permitted. You may ask for log tables, if required.*

Section A

1. If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. Find $A^{-1}A$

2.. Find the value of λ so that the vectors $\hat{\imath}$, $8\hat{\imath} - \lambda\hat{\jmath} - 3\hat{k}$ and $9\hat{\imath} - \hat{\jmath} - \hat{k}$ are coplanar.

3. Write the projection of vector $2\hat{\imath} + \hat{\jmath}$ on the vector $3\hat{\imath} - 4\hat{k}$

4. Find the angle between the line $\vec{r} = 2\hat{\imath} - 8\hat{\jmath} - 11\hat{k} + \mu(\hat{\imath} - \hat{k})$ and the plane $x - y + 10 = 0$

5. Determine the order and degree of the differential equation $\left(1 + \frac{dy}{dx}\right)^{\frac{2}{3}} = \frac{d^2y}{dx^2}$

6. Write the solution of the differential equation $\frac{dy}{dx} = 3^{-y}$

SECTION B

7. Prove that : $cos^{-1}\frac{12}{13} + sin^{-1}\frac{3}{5} = tan^{-1}\frac{56}{33}$

OR

Prove that $tan^{-1}1 + tan^{-1}2 + tan^{-1}3 = \pi$

8. If none of a, b and c is zero using the properties of dertminants prove that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

9. Show that the function f(x) = $|x-4|$, x ∈ R is continuous but not differentiable x = 4.

OR

If x = a (cos t + t sin t) and y = a (sin t – t cos t), find $\frac{d^2y}{dx^2}$

10. If y = $(tan^{-1}\frac{y}{x})^2$, show that $(x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$

11. Find the equation of tangent and normal to the curve y = $x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point (0, 5).

12. A whole sale shop has five dozen T – shirts with ' BE DISCIPLINE' printed three dozen with T – shirts 'BE PUNCTUAL' printed and six dozen T – shirts with 'BE HONEST' printed. The cost of each T – shirt is Rs 600, Rs 700 and Rs 800 respectively. All these items were sold in a day. Find the total amount of money collected by the shop by using matrix method. Which T – shirt would you like to buy and why?

13. Show that A = $\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = 0$. Hence, find A^{-1}.

OR

If A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$, find A^{-1}, using elementary row operations.

14. Evaluate : $\int \frac{x-5}{(x-3)^3} e^x \, dx$

15. Evaluate : $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} \, dx$

16. Evaluate : $\int \frac{1-x^2}{x-2x^2} \, dx$

OR

Evaluate the integral $\int_1^3 (3x^2 + 2x) \, dx$ as limit of sum

17. Find the shortest distance between the following

$\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z-3}{2}$; $\frac{x-4}{2} = \frac{y-5}{3} = \frac{z-6}{1}$

18. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = -9$

19. The probability that a student is not a swimmer is $\frac{1}{5}$. What is the probability that out of five students, four are swimmers? Do you like swimming? What are the advantages of swimming?

SECTION C

20. Let f: $W \to W$ be a function defined by $f(n) = \begin{cases} n-1, if\ n\ is\ odd \\ n+1, if\ n\ is\ even \end{cases}$, then show that f is invertible and hence find f^{-1}

21. In a country there are three states A, B and C. In state A 60% believe in honesty, while in state B, 70% and in state C, 80%. A person is selected at random from country and found that he is honest. Find the probability that he belongs to state B. Is honest person free from corruption? Justify your answer.

22. A factory two type of items A and B, made of plywood. One piece of item A requires 5 minutes for cutting and 10 minutes for assembling. One piece of item B requires 8 minutes for cutting and 8 minutes for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours for assembling. The profit on one piece of item A is Rs 5 and that on item B is Rs 6. How many pieces of each type should the factory make so as to maximize profit? Make it as an LPP and solve graphically.

23. An open tank with square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be least when the depth of the tank is half of its width.

OR

Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle 30^0 is $\frac{4}{81}\pi h^3$

24. Find the area of the region enclosed between the two circles:

$x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = 1$

OR

Using integration find the area of the region bounded by triangle ABC , where the vertices A, B and C are (-1. 1), (0, 5) and (3, 2) respectively.

25. Find the particular solution of the differential equation $x^2 dy + (xy + y^2)\, dx = 0$, y= 1when x =1

26. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (3, 2, 1) from the plane $2x - y + z + 1 = 0$. Also find the image of the point in the plane.