INTERNATIONAL INDIAN SCHOOL , DAMMAM
SECOND TERMINAL EXAMINATION 2015
GRADE – 12
SUBJECT : MATHEMATICS

TIME : 3 HOURS **Max. Marks : 100**

SET – A

General Instructions

1.All questions are compulsory.

2.The question paper consists of **26** questions divided into sections **A, B** and **C**. Section **A** comprises of **6** questions of one mark each, Section **B** comprises of **13** questions of four marks each and Section **C** comprises of **07** questions of six marks each.

3.All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4.There is no overall choice. However, internal choice has been provided in **04** questions of four marks each and **02** questions of six marks each. You have to attempt only one of the alternatives in all such questions.

5.This paper consists of FOUR printed pages.

SECTION A

1. Write the sum of the order and degree of the differential equation

$$\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0$$

2.For what value of k ,is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 4 \\ K & -4 & 0 \end{bmatrix}$ a skew-symmetric matrix?

3. Write the integrating factor of the differential equation

$(1 + y^2) + (2xy - \cot y)\frac{dy}{dx} = 0$

4.Write the unit vector perpendicular to both the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$

5 . Find the distance of a point **(2 , 5 , - 3)** from the plane $\vec{r} . (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$

6 . Find the value of **a + b** ,if the points **(2 , a , 3) , (3 , – 5 , b)** and **(– 1 , 11 , 9)** are collinear.

SECTION B

7. Find the value of ' k ' for which $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & , -1 \le x < 0 \\ \frac{2x+1}{x-1} & , 0 \le x < 1 \end{cases}$

is continuous at x = 0 .

8. If $y = e^{a \cos^{-1} x}$, $-1 \le x \le 1$,show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2 y = 0$$

OR

Verify Lagrange's Mean value theorem for the function $f(x) = x + \frac{1}{x}$ in $[1, 3]$.

9. If Show that $y = \log(1 + x) - \frac{2x}{2+x}$, $x > -1$ is an increasing function of x throughout its domain.

OR

If $y = \cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ find $\frac{dy}{dx}$.

10. A trust caring for handicapped children gets Rupees 30,000 every month from its donors. The trust spends half of the funds received for medical and educational care of the children and for that it charges 2% of the spent amount from them and deposits the balance amount in a private bank to get the money multiplied so that in future the trust goes on functioning regularly . What percent of interest should the trust get from the bank to get a total of Rupees 1,800 every month ?

Use matrix method ,to find the rate of interest .Do you think people should donate to such trust ?

11. Evaluate $\int(\sqrt{\cot x} + \sqrt{\tan x})\, dx$

12. Evaluate $\int(3 - 2x).\sqrt{2 + x - x^2}\, dx$

OR

$$\int \frac{\log x}{(x+1)^2}\, dx$$

13. Find the equations of the tangent and normal to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$.

14. Evaluate $\int \frac{x^2+1}{(x^2+4)(x^2+25)}\,dx$

OR

Evaluate ,using limit of sums;

$$\int_1^4 (x^2 - x)\ dx$$

15. Form the differential equation representing the family of ellipses having foci on x – axis and centre at origin.

16. Find the vector and Cartesian equations of a line through the point (1 ,-1 , 1) and perpendicular to the lines joining the points (4, 3 , 2), (1 ,-1 ,0) and (1 , 2 ,-1) ,(2 , 1 , 1) .

17. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ show that $\vec{a}-\vec{b}, \vec{b}-\vec{c}$ and $\vec{c}-\vec{a}$ are coplanar.

18. Prove that ,using properties of determinants

$$\begin{vmatrix} X^2+1 & XY & XZ \\ XY & Y^2+1 & YZ \\ ZX & ZY & Z^2+1 \end{vmatrix} = 1 + X^2 + Y^2 + Z^2.$$

19 . Find the shortest distance between the lines whose equations are

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$.

SECTION C

20. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C . Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C . It costs rupees 5 per kg to purchase Food I and rupees 7 per kg to purchase Food II . Determine the minimum cost of such a mixture .Formulate the above as a LPP and solve it .Do you agree that the nutrients are needful for good health ?

21. Find the area of the part of the circle $2x^2 + 2y^2 = 32$ which is exterior to the parabola $y^2 = 6x$

22. A school wants to award its students for the values honesty, regularity and hard work with a total cash award of Rupees 6000.Three times the award money for hardwork added to that given for honesty amounts to rupees 11000.The award money given for honesty and hardwork together is double the one given for regularity. Represent the above situation algebrically and find the award money for each value ,using matrix method.Apart from these values suggest one more value which the school must include for awards.

OR

Find the inverse of the following matrix using elementry operations.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

23. Show that the height of the cylinder of maximum volume that can be inscribed in a right circular cone of height h and semi vertical angle β is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \beta$.

OR

Show that semi- vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}(1/3)$.

24. Show that the differential equation $2ye^{x/y}dx + (y - 2xe^{x/y})\,dy = 0$ is homogeneous . Find the particular solution of this equation ,given that x = 0 when y = 1.

25. Find the value of k for which the following lines are perpendicular to each other ;

$$\frac{x+3}{k-5} = \frac{y-1}{1} = \frac{5-z}{-2k-1} \text{ and } \frac{x+2}{-1} = \frac{2-y}{-k} = \frac{z}{5}.$$

Hence ,find the equation of the plane containing the above lines .

26. Evaluate ,*using properties of definite integral* , $\int_0^{\pi} \boldsymbol{log(1+cos\,x)}\ \mathbf{dx}$
