INTERNATIONAL INDIAN SCHOOL – DAMMAM

FIRST TERMINAL EXAMINATION JULY- 2017

SUB: MATHEMATICS
CLASS: XII

TIME: 3 Hrs
MAX MARKS: 100

SET – B

Instructions:

i) All questions are compulsory

ii) The question paper consists of 29 questions divided into three sections A, B, C & D.

Section A comprises of 4 questions of 1 mark each.

Section B comprises of 8 questions of 2 marks each

Section C comprises of 11 questions of 4 marks each and

Section D comprises of 6 questions of 6 marks each.

iii) There is no overall choice. However internal choice has been provided in 3 questions of 4 marks each and 2 questions of 6 mark each. You have to attempt only one of the alternatives in all such questions.

iv) Use of calculator is not permitted.

Section – A

1. Show that if f : A → B and g : B → C are one-one then g o f : A→ C is also one – one.

2. Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

3. Compute the indicated products $\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}2 & 3 & 4\end{bmatrix}$.

4. Find x if $\begin{vmatrix}x & 2\\18 & x\end{vmatrix} = \begin{vmatrix}6 & 2\\18 & 6\end{vmatrix}$

Section – B

5. Find the inverse of $f(x) = \frac{x-1}{x+1}, x \neq -1$

6. Verify mean value theorem for the function $f(x) = x^2 - 6x + 5$, $x \in [-2, 3]$.

7. A particle move along the curve $6y = x^3 + 2$.Find the points on the curve at which the y coordinate is changing 8 times as fast as the x- coordinate.

8. Find the equation of the line joining A (1 , 3) and B (0 , 0) using determinants and find k if D(k , 0) is a point such that area of triangle ABD is 3 sq units.

9. Differentiate $\tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$ with respect to x.

10. If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,find k so that $A^2 = kA - 2I$

11. Prove that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$.

12. If f : R → R and g: R→ R are defined respectively as f (x) = $x^2 + 3x + 1$ and g (x) = 2x – 3, find f o g and g o f

<u>Section – C</u>

13. If $y = x\log\left(\frac{x}{a+bx}\right)$ prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

14. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$, $-\frac{1}{\sqrt{2}} \leq x \leq 1$

15. . Find all points of discontinuity of the function f defined by $f(x) = \begin{cases} x+2, & x \leq 1 \\ x-2, & 1 < x < 2 \\ 0, & x \geq 2 \end{cases}$

16. Express $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix

17. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect to x.

18. Solve for x : $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}$,($-1 < x < 1$).

19. If F(x) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$,show that F(x) . F(y) = F(x+y)

OR

If A' = $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$,then verify that (A + B)' = A' + B' .

20. Using property of determinants prove that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

21. Given a non empty set X, let $* : P(X) \times P(X) \to P(X)$ be defined as $A * B = (A - B) \cup (B - A)$ for all $A, B \in P(X)$.Show that the empty set $\emptyset$ is the identity for the operation $*$ and all the

elements A of $P(X)$ are invertible with $A^{-1} = A$

OR

Consider the binary operations $* : R \times R \to R$ and $o : R \times R \to R$ defined as $a * b = |a - b|$ and , $a \, o \, b = a$ for all a , b $\in R$ .Show that $*$ is commutative but not associative , o is associative but not commutative.

22. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$ find $\frac{d^2y}{dx^2}$.

OR

Differentiate $e^{(x^3+\frac{1}{2}\log x)}$ with respect to x.

23. Prove that the relation R on the set N × N defined by (a , b) R (c , d) <=> a + d = b + c for all (a , b) , (c, d) ∈ N × N is an equivalence relation .

Section – D

24. For the curve $y = 4x^3 - 2x^5$,find all the points at which the tangent passes through the origin.
25. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere

OR

Show that the right circular cylinder of given surface area and maximum volume is such that its height is equal to the diameter of the base.

26. Using elementary transformations , find the inverse of $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$
27. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is

    a) strictly increasing b) strictly decreasing.
28. Two schools P and Q want to award their selected students on the values of discipline, politeness, and punctuality. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to its 3, 2 & 1 student with a total award money of Rs 1000.

School Q wants to spend Rs 1500 to award it's 4, 1 & 3 students on the respective values (by giving the same award money for the three values as before).If the total amount of awards for one prize on each value is Rs 600, using matrices, find the award money for each value. Apart from the above three values, suggest one more value for awards.

29. If x and y are connected parametrically by the equation $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$, $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, prove that $\frac{dy}{dx} = -\cot 3t$.

OR

Differentiate with respect to x, $x^{x^2-3} + (x-3)^{x^2}$ for $x > 3$