

INTERNATIONAL INDIAN SCHOOL – DAMMAM
FIRST TERMINAL EXAMINATION JUNE - 2014

GRADE XI

Subject: Mathematics

Max marks: 100

Time: 3 hours

SET A

General Instructions:

1. All questions are compulsory.
2. The question paper consists of 26 questions divided into three sections A, B and C
3. Section A contains 6 questions of 1 mark each, Section B contains 13 questions of 4 marks each, Section C contains 7 questions of 6 marks each.
4. There is no overall choice in the paper. However, internal choice is provided in 3 questions of 4 mark each and 2 questions of 6 mark each.
5. Use of calculators is not permitted.

SECTION A

Questions from 1 to 6 carry 1 mark each

1. If the power set of set A have 64 elements, then how many elements does A have ?
2. If $P = \{1, 2\}$, find $P \times P \times P$.
3. Find the multiplicative inverse of $3 - 2i$.
4. Find the value of $\tan \frac{19\pi}{3}$
5. If A and B are any two sets such that $B \subset A$, then find $B \cap A$.
6. Find the principal solutions of $\tan x = -\sqrt{3}$

SECTION B

Questions from 7 to 19 carry 4 marks each

7. Find the square root of $-5 - 12i$.
8. Find the general solutions of $\sin 6x + \sin 2x - \sin 4x = 0$
9. If $U = \{x : x \in N, x \leq 10\}$ is the universal set for $A = \{1, 2, 5, 6\}$ and $B = \{x : x^2 - 13x - 42 = 0\}$ then verify that
 - (i) $A - B = B' - A'$
 - (ii) $A' \cup B = (A \cap B)'$

10. Find the domain and range of the functions

(i) $f(x) = \sqrt{x-1}$

(ii) $f(x) = -|x|$

11. If $(x + iy)^3 = u + iv$ then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

OR

If $a + ib = \frac{(x + i)^2}{2x^2 + 1}$ Prove that $a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$

12. If $\sin x = \frac{4}{5}$, where x is in IInd quadrant, find $\sin \frac{x}{2}$, $\cos \frac{x}{2}$, $\tan \frac{x}{2}$.

13. If $P(A) = P(B)$, Show that $A = B$

OR

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. A relation R from A to A is defined by $R = \{(x, y) : y = x + 2\}$

(i) Write R in Roster form.

(ii) Depict this relation using an arrow diagram.

(iii) Write down the domain and range of R .

14. Prove that $\frac{1 + \sin 2\theta + \cos 2\theta}{1 + \sin 2\theta - \cos 2\theta} = \cot \theta$

15. Using the principle of mathematical induction prove that

$$3^{2n+2} - 8n - 9 \text{ is divisible by } 8.$$

OR

Prove by the principle mathematical induction that $41^n - 14^n$ is a multiple of 27.

16. A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

17. If $A = \{1, 2, 3\}$, $B = \{2, 4\}$, $C = \{2, 6\}$. Verify that

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

18. Prove that $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \left(\frac{x+y}{2} \right)$

OR

Show that $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$

19. Express $\frac{1+3i}{1-2i}$ in polar form.

SECTION C

Questions from 20 to 26 carry 6 marks each

20. Solve graphically the equations

$$x + 2y \leq 6, 2x + y \geq 2, x - y \leq 1, x \geq 0, y \geq 0$$

21. In a class of 60 students, 23 play hockey, 15 play basket ball and 20 play cricket, 7 play hockey and basket ball, 5 play cricket and basket ball, 4 play hockey and cricket and 15 students do not play any of these games. Find (i) How many play hockey, basket ball and cricket? (ii) How many play hockey but not cricket? (iii) How many play hockey and cricket but not basket ball? (iv) Suggest any two values which the students will acquire by taking part in different games.

22. Find real θ such that $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$ is purely real.

OR

Let $z_1 = 2 - i$ and $z_2 = -2 + i$, find (i) $\operatorname{Re} \left(\frac{z_1 z_2}{z_1} \right)$ and (ii) $\operatorname{Im} \left(\frac{1}{z_1 \bar{z}_1} \right)$

23. Prove that $\cos^2 x + \cos^2 \left(x + \frac{\pi}{3} \right) + \cos^2 \left(x - \frac{\pi}{3} \right) = \frac{3}{2}$

24. Prove by the principle of mathematical induction

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

25. Prove that $(b+c) \cos \left(\frac{B+C}{2} \right) = a \cos \left(\frac{B-C}{2} \right)$

OR

Prove that $(b^2 - c^2) \cot A + (c^2 - a^2) \cot B + (a^2 - b^2) \cot C = 0$

26. (a) Draw the graph of the function defined by $f(x) = x^2 + 1$

(b) Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ be a linear function from Z to Z defined by $f(x) = mx + c$. Find $f(x)$.
